Problem Set: Probability

**When solving the following problems, please clearly specify what you try to do step by step whenever necessary. Here is an example: finding the mode of 1, 0, -1, 1, -1, 2.**

**Step 1: rearrange all the numbers in ascending order.**

**-1, -1, 0, 1, 1, 2**

**Step 2: count the frequency of each unique number**

**-1: 2**

**0: 1**

**1: 2,**

**2: 1**

**Step 3: determine the mode**

**The mode of this sequence of numbers are -1 and 1.**

1. Consider a random experiment in which we roll two fair, six-sided dices. Let random variable X be the sum of the two numbers resulted from a roll. Answer the following questions.
2. Let A be the event that X is even. What is the value of P(A)?

Sample space of A: X = {2,4,6,8,10, 12}

Occurrence of A: 1,3,5,5,3,1 respectively

P(A) = 18 / 36 = 0.5

1. Let B be the event that X. Compute P(B), P(AB) and P(AB). .41 0.25,0.75

Step: 1 Calculate probability of event B

Sample space of B: X = {8,9,10,11,12}

Occurrence of B: 5,4,3,2,1 respectively

**P(B) = 15 / 36 = 0.41**

Step: 2 Calculate probability of event P(AB)

sample space: X = {8, 10, 12}

Occurrence of B: 5,3,1 respectively

**P(AB) = 9 / 36 = 0.25**

Step: 3 Calculate probability of event P(AB)

sample space: X = {2,4,6,8,9,10,11,12}

Occurrence of B: 1,3,5,5,4,3,2,1 respectively

**P(AB) = 24 / 36 = 0.667**

1. Let C be the event that . Compute P(C), P(AC) and P(AC).

Step: 1 Calculate probability of event C

Sample space of B: X = {6,7,8,9 }

Occurrence of B: 5,6,5,4 respectively

**P(C) = 20 / 36 = 0.556**

Step: 2 Calculate probability of event P(AC)

sample space: X = {6,8}

Occurrence of B: 5,5 respectively

**P(AC) = 10 / 36 = 0.2778**

Step: 3 Calculate probability of event P(AC)

sample space: X = {2,4,6,7,8,9,10,12}

Occurrence of B: 1,3,5,6,5,4,3,1 respectively

**P(AC) = 28 / 36 = 0.7778**

1. Are events A and B independent? Why or why not?

No. Since sum 8 will be a common occurrence in both events A and B

1. Are events A and C independent? Why or why not?

No. Since sum 8 will be a common occurrence in both events A and C

1. Please use the tabular approach (joint probability table) first and then Bayes’ Theorem to answer problem 42 on page 207 of the textbook.

Question: 2

A local bank reviewed its credit card policy with the intention of recalling some of its credit cards. In the past, approximately 5% of cardholders defaulted, leaving the bank unable to collect the outstanding balance. Hence, management established a prior probability of .05 that any cardholder will default. The bank also found that the probability of missing a monthly payment is .20 for customers who do not default. Of course, the probability of missing a monthly payment for those who default is 1.

* 1. Given that a customer missed one or more monthly payments, compute the posterior probability that the customer will default.
  2. The bank would like to recall its card if the probability that a customer will default is greater than .20. Should the bank recall its card if the customer misses a monthly payment? Why or why not?

Step: 1 Let’s assume

M = Missed payments

D = Customer defaults

D1 = Customer does not default

Step: 2 Using Joint probability table for customer default and missing monthly payments

|  |  |  |  |
| --- | --- | --- | --- |
|  | Default customer  P(D) | Non-Default customer  P(D1) | Total |
| Making monthly payment  P(M) | 0.05 | 0.19 | 0.24 |
| Missing monthly payment  P(M1) | 0 | 0.76 | 0.76 |
| Total | 0.05 | 0.95 | 1 |

P(D) = 0.05, P(D1) = 0.95, P(M|D1) = 0.2, P(M|D) = 1

P(D|M) = {P(D) P(M|D)} / {P(D) P(M|D) + P(D1) P(M|D1)}

= {(0.05) (1)} / {(0.05) (1) + (0.95) (0.2)}

= (0.05) / (0.24)

= 0.21

Using Bayes’ Theorem:

Let B1, B2, ……, Bn be a partition of sample space and A be another event

P(Bi|A) = { P(A|Bi)/ P(Bi) } / {P(B1) P(A|B1) + P(B2) P(A|B2) + ……+ P(Bn) P(A|Bn)}

P(D|M) = {P(D) P(M|D)} / {P(D) P(M|D) + P(D1) P(M|D1)}

= {(0.05) (1)} / {(0.05) (1) + (0.95) (0.2)}

= (0.05) / (0.24)

= 0.21